

Some additional exercise questions of stochastic calculus

March 16, 2024

Throughout the exercise questions, we use $(B_t)_{t \geq 0}$ to denote a standard Brownian motion.

Week 9.

- If the process $M_t := B_t^3 - \int_0^t f(B_s) ds$ is a martingale. Write down the function form of f , and express M_t in the form of an Itô integral.
- If $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a deterministic continuous function. Show that $\int_0^t f(s) dB_s$ follows a normal distribution, and compute its mean and variance.

Week 10.

- If the process $M_t := t^2 B_t^2 - \int_0^t f(s, B_s) ds$ is a martingale. Write down the function form of f , and express M_t in the form of an Itô integral.
- Let $Y_t = B_t \cdot \int_0^t B_s dB_s$. Compute dY_t .