

In the second condition satisfied by the function  $f$

$$\mathbf{P}f(x) = g(x), \quad \text{for } x \in S \setminus A.$$

Should be

$$\mathbf{P}f(x) - f(x) = g(x), \quad \text{for } x \in S \setminus A.$$